Statistics 251
Section 2, Autumn 2020
Midterm Exam
October 26, 2020
Time Limit: 50 Minutes

Name: ____________________________

CNetID: ____________________________

Instructions: This exam contains 4 problems. Please make sure you attempt all problems.

Present your solutions in a **legible, coherent** manner. Unless otherwise specified, you should show your work; you will be evaluated on both your reasoning and your answer. Unsupported or illegible solutions may not receive full credit.

Please write your **final answer** for each problem in the provided box. Please show your work in the space below the box. If you need additional space for scratchwork, you may use the blank pages stapled to the end of the exam. Please **do not write on the back side of pages**.

You will have 50 minutes to complete this exam. You may choose any 50 minute period between 10:30am and 12 midnight Chicago time on October 26, 2020 to take the exam. You must scan and upload the completed exam to Gradescope by **12 midnight Chicago time on October 26, 2020**. Please write the 50 minute period you took the exam below.

Start Time: ____________________________

End Time: ____________________________

The use of outside material including books, notes, calculators, and electronic devices is not allowed. Due to the coronavirus situation, this exam will be take-home. Please sign below to affirm that you have followed these rules.

Signature: ____________________________

Formulas

Probability mass functions:

- Binomial with parameters n and p: $\mathbb{P}(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$
- Poisson with parameter λ: $\mathbb{P}(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}$
- Geometric with parameter p: $\mathbb{P}(X = k) = p(1-p)^{k-1}$
- Negative binomial with parameters r and p: $\mathbb{P}(X = k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}$

Problem 1 (20 points) A class of 6 seniors and 6 juniors is to be placed into 4 Zoom breakout rooms, each with 3 students.

(a) (10 points) How many ways can this be done, if the rooms are not distinguishable?

Answer:

(b) (10 points) How many ways can this be done, if the rooms are not distinguishable and each room must have at least one senior?

Answer:

Problem 2 (20 points) Let E, F, G be events in a probability space with sample space S. Suppose that $\mathbb{P}(E) = 0.3$ and $\mathbb{P}(F) = 0.8$.

(a) (5 points) What are the minimum and maximum possible values of $\mathbb{P}(E \cup F)$?

Answer:

(b) (5 points) What are the minimum and maximum possible values of $\mathbb{P}(E \cap F)$?

Answer:

(c) (10 points) We say that E and F are independent conditioned on G if $\mathbb{P}(E \cap F \mid G) = \mathbb{P}(E \mid G)\mathbb{P}(F \mid G)$. If E and F are independent conditioned on G and independent conditioned on G^c, is it necessarily the case that E and F are independent? If your answer is yes, explain why; if your answer is no, give a counterexample.

Answer:

Problem 3 (20 points) Bob buys a lottery ticket. Lottery tickets come in two color, blue and red. Blue tickets have a $\frac{1}{100}$ chance of winning, and red tickets have a $\frac{1}{1000}$ chance of winning. When buying a ticket, the buyer gets a blue ticket with probability $\frac{1}{2}$ and a red ticket with probability $\frac{1}{2}$.

(a) (10 points) If Bob buys a lottery ticket, what is the probability that it wins?

Answer:

(b) (10 points) Given that Bob's single ticket won, what is the probability that it is blue?

Answer:

Problem 4 (40 points) Alice is searching for needles in a haystack containing many needles. She rummages through the haystack every 5 minutes and has probability $\frac{1}{120}$ of finding a needle each time.

(a) (5 points) Compute the expected amount of time before Alice finds the first needle. (If Alice finds a needle during a 5-minute period, we compute the elapsed time at the end of the period.)

Answer:

(b) (5 points) Compute the expected amount of time before Alice finds the 10^{th} needle. (If Alice finds a needle during a 5-minute period, we compute the elapsed time at the end of the period.)

Answer:

(c) (10 points) Let X be the number of needles Alice finds in a 10 hour period. Compute $\mathbb{E}[X]$ and $\mathrm{Var}(X)$.

Answer:

(d) (10 points) Let X be the number of needles Alice finds in a 10 hour period. Compute $\mathbb{P}(X \geq 2)$, the probability that Alice finds at least 2 needles in a 10 hour period.

Answer:

(e) (10 points) Using a Poisson approximation for X, approximate the probability $\mathbb{P}(X = 4)$.

Answer: