Statistics 251
Section 2, Autumn 2020
Practice Midterm
October 26, 2020
Time Limit: 50 Minutes

Name: ____________________________

CNetID: ____________________________

Instructions: This exam contains 4 problems. Please make sure you attempt all problems.

Present your solutions in a **legible, coherent** manner. Unless otherwise specified, you should show your work; you will be evaluated on both your reasoning and your answer. Unsupported or illegible solutions may not receive full credit.

Please write your **final answer** for each problem in the provided box. Please show your work in the space below the box. If you need additional space for scratchwork, you may use the blank pages stapled to the end of the exam. Please **do not write on the back side of pages**.

You will have 50 minutes to complete this exam. You may choose any 50 minute period between 10:30am and 12 midnight Chicago time on October 26, 2020 to take the exam. You must scan and upload the completed exam to Gradescope by **12 midnight Chicago time on October 26, 2020**. Please write the 50 minute period you took the exam below.

Start Time: ____________________________

End Time: ____________________________

The use of outside material including books, notes, calculators, and electronic devices is not allowed. Due to the coronavirus situation, this exam will be take-home. Please sign below to affirm that you have followed these rules.

Signature: ____________________________

Formulas

Probability mass functions:

- Binomial with parameters n and p: $\mathbb{P}(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$
- Poisson with parameter λ: $\mathbb{P}(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}$
- Geometric with parameter p: $\mathbb{P}(X = k) = p(1-p)^{k-1}$
- Negative binomial with parameters r and p: $\mathbb{P}(X = k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}$

Problem 1 (10 points) How many quintuples $(a_1, a_2, a_3, a_4, a_5)$ of non-negative integers satisfy $a_1 + a_2 + a_3 + a_4 + a_5 = 100$?

Answer:

Problem 2 (20 points) Thirty people are invited to a party. Each person accepts the invitation, independently of all others, with probability 1/3. Let X be the number of accepted invitations. Compute the following:

(a) (5 points) $\mathbb{E}[X]$

Answer:

(b) (5 points) $\mathrm{Var}(X)$

Answer:

(c) (5 points) $\mathbb{E}[X^2]$

Answer:

(d) (5 points) $\mathbb{E}[X^2 - 4X + 5]$

Answer:

Problem 3 (20 points) Bob has noticed that during every given minute, there is a 1/720 chance that the Facebook page for his dry cleaning business will get a like, independently of what happens during any other minute. Let L be the total number of likes that Bob receives during a 24 hour period.

(a) (5 points) Compute $\mathbb{E}[L]$ and $\mathrm{Var}(X)$.

Answer:

(b) (5 points) Compute the probability that $L = 0$.

Answer:

(c) (10 points) Use a Poisson approximation to approximate the probability that $L \geq 2$.

Answer:

Problem 4 (10 points) Consider an infinite sequence of independent tosses of a coin that comes up heads with probability p. Compute (in terms of p) the probability that the fifth head occurs on the tenth toss.

Answer:

Problem 5 (20 points) A standard deck of 52 cards contains 4 aces. Suppose we choose a random ordering (all 52! permutations being equally likely). Compute the following:

(a) (5 points) The probability that all of the top 4 cards in the deck are aces.

Answer:

(b) (5 points) The probability that none of the top 4 cards in the deck is an ace.

Answer:

(c) (10 points) The expected number of aces among the top 4 cards in the deck.

Answer:

Problem 6 (20 points) There are ten children: five attend school A, three attend school B, and two attend school C. Suppose that a pair of two children is chosen uniformly at random from the set of all possible pairs of children. Let X be the number of students in the random pair that attend school A and let Y be the number in the pair that attend school B.

(a) (10 points) Compute $\mathbb{E}[XY]$.

Answer:

(b) (10 points) Given that the two children in this pair attend the same school, what is the conditional probability that they both attend school A?

Answer: