Statistics 251, Autumn 2020 — Homework 1

Due date: 11:30am on Monday, October 5, 2020 on Gradescope.

Instructions: Please present your solutions in a legible, coherent manner. Unless otherwise specified, you should show your work; you will be evaluated on both your reasoning and your answer. Points may be deducted for unclear or messy solutions.

Collaboration and Academic Integrity: You are encouraged to collaborate on homework. However, you must write your solutions alone and **understand what you write**. When submitting your homework, list in the space below any sources you used (in print, online, or human) other than the textbook or the teaching staff.

0. [0pts] What time zone are you currently in? Do you plan to attend lectures synchronously (during the usual class time via Zoom), or watch the recorded lectures asynchronously on your own time?
1. [10pts] Consider a group of 20 people. If everyone shakes hands with everyone else, how many handshakes take place?
2. [10pts] A dance class consists of 22 students, of which 10 are women and 12 are men. If 5 men and 5 women are to be chosen and then paired off, how many results are possible?
3. [10pts] From a group of 8 women and 6 men, a committee consisting of 3 men and 3 women is to be formed. How many different committees are possible if
   a. 2 of the men refuse to serve together?
   b. 2 of the women refuse to serve together?
   c. 1 man and 1 woman refuse to serve together?
4. [10pts] If 8 identical blackboards are to be divided among 4 schools, how many divisions are possible? How many if each school must receive at least 1 blackboard?
5. [10pts] For any sequence of events $E_1, E_2, \ldots$, define a new sequence $F_1, F_2, \ldots$ of disjoint events such that for all $n \geq 1$,
$$\bigcup_{i=1}^{n} F_i = \bigcup_{i=1}^{n} E_i.$$
6. [10pts] If $\mathbb{P}(E) = 0.9$ and $\mathbb{P}(F) = 0.8$, show that $\mathbb{P}(E \cap F) \geq 0.7$. In general, prove Bonferroni's inequality, namely
$$\mathbb{P}(E \cap F) \geq \mathbb{P}(E) + \mathbb{P}(F) - 1.$$
7. [10pts] In an experiment, a die is rolled continually until a 6 appears, at which point the experiment stops. What is the sample space of this experiment? Let E_n denote the event that n rolls are necessary to complete the experiment. What points of the sample space are contained in E_n? What is $\left(\bigcup_{n=1}^{\infty} E_n\right)^c$?
8. [10pts] A, B, and C take turns flipping a coin. Assume that A flips first, then B, then C, then A, and so on. The first one to get a head wins. The sample space of this experiment can be defined by
$$S = \{H, TH, TTH, TTTH, \ldots\}.$$
   a. Interpret the sample space.
   b. Define the following events in terms of S:
      i. A wins $= X$;
      ii. B wins $= Y$;
      iii. $(X \cup Y)^c$.