Due date: 11:30am on Monday, October 12, 2020 on Gradescope.

Instructions: Please present your solutions in a legible, coherent manner. Unless otherwise specified, you should show your work; you will be evaluated on both your reasoning and your answer. Points may be deducted for unclear or messy solutions.

Collaboration and Academic Integrity: You are encouraged to collaborate on homework. However, you must write your solutions alone and **understand what you write**. When submitting your homework, list in the space below any sources you used (in print, online, or human) other than the textbook or the teaching staff.

1. [10pts] Prove Boole's inequality

$$\mathbb{P}\left(\bigcup_{i=1}^{\infty} A_i\right) \leq \sum_{i=1}^{\infty} \mathbb{P}(A_i).$$

2. [10pts] Consider an experiment whose sample space consists of a countably infinite number of points. Show that not all points can be equally likely. Can all points have a positive probability of occurring?

3. [10pts] An urn contains n red and m blue balls. They are withdrawn one at a time until a total of r, $r < n$, red balls have been withdrawn. Find the probability that a total of k balls are withdrawn.
   Hint: A total of k balls will be withdrawn if there are $r - 1$ red balls in the first $k - 1$ withdrawals and the rth withdrawal is a red ball.

4. [10pts] Consider 3 urns. Urn A contains 2 white and 4 red balls, urn B contains 8 white and 4 red balls, and urn C contains 1 white and 3 red balls. If 1 ball is selected from each urn, what is the probability that the ball chosen from urn A was white given that exactly 2 white balls were selected?

5. [10pts] An ectopic pregnancy is twice as likely to develop when the pregnant woman is a smoker as it is when she is a nonsmoker. If 32 percent of women of childbearing age are smokers, what percentage of women having ectopic pregnancies are smokers?

6. [10pts] Ms. Aquina has just had a biopsy on a possibly cancerous tumor. Not wanting to spoil a weekend family event, she does not want to hear any bad news in the next few days. But if she tells the doctor to call only if the news is good, then if the doctor does not call, Ms. Aquina can conclude that the news is bad. So, being a student of probability, Ms. Aquina instructs the doctor to flip a coin. If it comes up heads, the doctor is to call if the news is good and not call if the news is bad. If the coin comes up tails, the doctor is not to call. In this way, even if the doctor doesn't call, the news is not necessarily bad. Let α be the probability that the tumor is cancerous; let β be the conditional probability that the tumor is cancerous given that the doctor does not call.
   a. Which should be larger, α or β?
   b. Find β in terms of α, and prove your answer in part (a).

7. [10pts] In a class, there are 4 first-year boys, 6 first-year girls, and 6 sophomore boys. How many sophomore girls must be present if sex and class are to be independent when a student is selected at random?

8. [10pts] A true-false question is to be posed to a husband-and-wife team on a quiz show. Both the husband and the wife will independently give the correct answer with probability p. Which of the following is a better strategy for the couple?
   a. Choose one of them and let that person answer the question.
   b. Have them both consider the question, and then either give the common answer if they agree or, if they disagree, flip a coin to determine which answer to give.