

## Statistics 251, Autumn 2020 — Homework 3

Due date: 11:30am on Monday, October 19, 2020 on Gradescope.

**Instructions:** Please present your solutions in a legible, coherent manner. Unless otherwise specified, you should show your work; you will be evaluated on both your reasoning and your answer. Points may be deducted for unclear or messy solutions.

**Collaboration and Academic Integrity:** You are encouraged to collaborate on homework. However, you must write your solutions alone and **understand what you write**. When submitting your homework, list in the space below any sources you used (in print, online, or human) other than the textbook or the teaching staff.

- [10pts] Five distinct numbers are randomly distributed to players numbered 1 through 5. Whenever two players compare their numbers, the one with the higher one is declared the winner. Initially, players 1 and 2 compare their numbers; the winner then compares her number with that of player 3, and so on. Let  $X$  denote the number of times player 1 is a winner. Find  $\mathbb{P}(X = i)$  for  $i = 0, 1, 2, 3, 4$ .
- [10pts] You have \$1000, and a certain commodity presently sells for \$2 per ounce. Suppose that after one week the commodity will sell for either \$1 or \$4 an ounce, with these two possibilities being equally likely.
  - If your objective is to maximize the expected amount of money that you possess at the end of the week, what strategy should you employ?
  - If your objective is to maximize the expected amount of the commodity that you possess at the end of the week, what strategy should you employ?
- [10pts] An insurance company writes a policy to the effect that an amount of money  $A$  must be paid if some event  $E$  occurs within a year. If the company estimates that  $E$  will occur within a year with probability  $p$ , what should it charge the customer in order that its expected profit will be 10 percent of  $A$ ?
- [10pts] A sample of 3 items is selected at random from a box containing 20 items of which 4 are defective. Find the expected number of defective items in the sample.
- [10pts] Each night different meteorologists give us the probability that it will rain the next day. To judge how well these people predict, we will score each of them as follows: If a meteorologist says that it will rain with probability  $p$ , then he or she will receive a score of

$$1 - (1 - p)^2 \text{ if it does rain}$$

$$1 - p^2 \text{ if it does not rain.}$$

We will then keep track of scores over a certain time span and conclude that the meteorologist with the highest average score is the best predictor of weather. Suppose now that a given meteorologist is aware of our scoring mechanism and wants to maximize his or her expected score. If this person truly believes that it will rain tomorrow with probability  $p^*$ , what value of  $p$  should he or she assert so as to maximize the expected score?

- [10pts] To determine whether they have a certain disease, 100 people are to have their blood tested. However, rather than testing each individual separately, it has been decided first to place the people into groups of 10. The blood samples of the 10 people in each group will be pooled and analyzed together. If the test is negative, one test will suffice for the 10 people, whereas if the test is positive, each of the 10 people will also be individually tested and, in all, 11 tests will be made on this group. Assume that the probability that a person has the disease is 0.1 for all people, independently of one another, and compute the expected number of tests necessary for each group. (Note that we are assuming that the pooled test will be positive if at least one person in the pool has the disease.)
- [10pts] A box contains 5 red and 5 blue marbles. Two marbles are withdrawn randomly. If they are the same color, then you win \$1.10; if they are different colors, then you win  $-\$1.00$ . (That is, you lose \$1.00.) Calculate
  - the expected value of the amount you win;
  - the variance of the amount you win.
- [10pts] Let  $X$  be a random variable having expected value  $\mu$  and variance  $\sigma^2$ . Find the expected value and variance of  $Y = \frac{X - \mu}{\sigma}$ .