Statistics 251, Autumn 2020 — Homework 4

Due date: 11:30am on Monday, October 26, 2020 on Gradescope.

Instructions: Please present your solutions in a legible, coherent manner. Unless otherwise specified, you should show your work; you will be evaluated on both your reasoning and your answer. Points may be deducted for unclear or messy solutions.

Collaboration and Academic Integrity: You are encouraged to collaborate on homework. However, you must write your solutions alone and **understand what you write**. When submitting your homework, list in the space below any sources you used (in print, online, or human) other than the textbook or the teaching staff.

1. [10pts] If X is a geometric random variable, prove that
$$\mathbb{P}(X = n + k \mid X > n) = \mathbb{P}(X = k).$$
2. [10pts] Let X be a negative binomial random variable with parameters r and p, and let Y be a binomial random variable with parameters n and p. Show that
$$\mathbb{P}(X > n) = \mathbb{P}(Y < r).$$
3. [10pts] It is known that diskettes produced by a certain company will be defective with probability 0.01, independently of one another. The company sells the diskettes in packages of size 10 and offers a money-back guarantee that at most 1 of the 10 diskettes in the package will be defective. The guarantee is that the customer can return the entire package of diskettes if he or she finds more than 1 defective diskette in it. If someone buys 3 packages, what is the probability that he or she will return exactly 1 of them?
4. [10pts] Approximately $80,000$ marriages took place in the state of New York last year. Estimate the probability that for at least one of these couples,
   a. both partners were born on April 30;
   b. both partners celebrated their birthday on the same day of the year.

   State your assumptions.
5. [10pts] Suppose that the number of accidents occurring on a highway each day is a Poisson random variable with parameter $X = 3$.
   a. Find the probability that 3 or more accidents occur today.
   b. Repeat part (a) under the assumption that at least 1 accident occurs today.
6. [10pts] The number of times that a person contracts a cold in a given year is a Poisson random variable with parameter $X = 5$. Suppose that a new wonder drug has just been marketed that reduces the Poisson parameter to $X = 3$ for 75 percent of the population. For the other 25 percent of the population, the drug has no appreciable effect on colds. If an individual tries the drug for a year and has 2 colds in that time, how likely is it that the drug is beneficial for him or her?
7. [10pts] A fair coin is continually flipped until heads appears for the 10th time. Let X denote the number of tails that occur. Compute the probability mass function of X.
8. [10pts] Suppose that a batch of 100 items contains 6 that are defective and 94 that are not defective. If X is the number of defective items in a randomly drawn sample of 10 items from the batch, find (a) $\mathbb{P}(X = 0)$ and (b) $\mathbb{P}(X > 2)$.