Statistics 251, Autumn 2020 — Homework 5

Due date: 11:30am on Monday, November 2, 2020 on Gradescope.

Instructions: Please present your solutions in a legible, coherent manner. Unless otherwise specified, you should show your work; you will be evaluated on both your reasoning and your answer. Points may be deducted for unclear or messy solutions.

Collaboration and Academic Integrity: You are encouraged to collaborate on homework. However, you must write your solutions alone and **understand what you write**. When submitting your homework, list in the space below any sources you used (in print, online, or human) other than the textbook or the teaching staff.

1. [10pts] Let X be a random variable with density function
$$f(x) = \begin{cases} c(1-x^2) & -1 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$
What is the value of c? What is the CDF of X?
2. [10pts] The lifetime in hours of an electronic tube is a random variable with probability density $f(x) = xe^{-x}$, $x \geq 0$. Compute the expected lifetime of such a tube.
3. [10pts] A point is chosen uniformly at random on a line segment of length L, dividing it into two segments. Find the probability that the ratio of the shorter to the longer is less than $\frac{1}{4}$.
4. [10pts] Suppose that X is a normal random variable with mean 5. If $\mathbb{P}(X > 9) = 0.2$, what is the approximate value of $\text{Var}(X)$?
5. [10pts] In 10000 independent tosses of a coin, the coin landed on heads 5800 times. Is it reasonable to assume that the coin is not fair? Explain.
6. [10pts] For some constant c, the random variable X has density function
$$f(x) = \begin{cases} cx^4 & 0 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$$
Find $\mathbb{E}[X]$ and $\text{Var}(X)$.
7. [10pts] Let X be a random variable that takes on values between 0 and c, meaning $\mathbb{P}(0 \leq X \leq c) = 1$. Show that
$$\text{Var}(X) \leq \frac{c^2}{4}.$$
8. [10pts] Let Z be a standard normal random variable, and let f be a differentiable function with derivative f'. Show that
$$\mathbb{E}[f'(Z)] = \mathbb{E}[Zf(Z)].$$