Mathematics UN1102
Section 3, Fall 2017
Practice Final 2
December 18, 2017
Time Limit: 170 Minutes

Name: ____________________________

UNI: ____________________________

Instructions: This exam contains 10 problems. Please make sure you attempt all problems.

Present your solutions in a **legible, coherent** manner. Unless otherwise specified, you should show your work; you will be evaluated on both your reasoning and your answer. Unsupported or illegible solutions may not receive full credit.

Please write your **final answer** for each problem in the provided box. Please show your work in the space below the box. If you need additional space for scratchwork, you may use the blank pages stapled to the end of the exam. **Do not write on the back side of your test papers.**

The use of outside material including books, notes, calculators, and electronic devices is not allowed.

Question	1	2	3	4	5	6	7	8	9	10	Total
Points	15	8	15	8	10	10	10	8	8	8	100
Score											

Formulas

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta$$

$$\sin(2\theta) = 2\sin\theta\cos\theta \qquad \cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\sin^2\theta = \frac{1}{2}(1 - \cos(2\theta)) \qquad \cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$$

$$\int \frac{1}{a^2 + x^2}dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C \qquad \int \frac{1}{\sqrt{a^2 - x^2}}dx = \arcsin\left(\frac{x}{a}\right) + C$$

$$\int \tan x\,dx = \ln|\sec x| + C \qquad \int \sec x\,dx = \ln|\sec x + \tan x| + C$$

Taylor series of $f(x)$ at $x = a$:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

Maclaurin series:

- $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots$ $\quad R = 1$
- $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ $\quad R = \infty$
- $\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ $\quad R = \infty$
- $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1}\frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ $\quad R = 1$
- $(1+x)^k = \sum_{n=0}^{\infty} \frac{k(k-1)\cdots(k-n+1)}{n!}x^n = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \cdots$ $\quad R = 1$

Problem 1 This problem is spread over two pages.

(a) (5 points) Evaluate

$$\int (3x+8)^{241}\,dx.$$

Answer:

(b) (5 points) Evaluate

$$\int \sin^3(\theta)\cos^{14}(\theta)\,d\theta.$$

Answer:

(c) (5 points) Evaluate

$$\int \frac{6x^2 - x + 18}{x(x^2 + 9)} dx.$$

Answer:

Problem 2 (8 points) Consider the region A bounded by the curves $y = x^4$ and $y = x$. Note that A lies entirely in the region $x \geq 0$.

(a) (2 points) Sketch the region A and find the coordinates of the two points where the curves intersect.

Answer:

(b) (6 points) Set up and evaluate an integral to find the volume of the solid of revolution obtained by rotating the region A about the x-axis.

Answer:

Problem 3 (15 points) Determine whether each of the following series is convergent or divergent. You do not need to include your work.

(a) (3 points) $\sum_{n=0}^{\infty} \frac{(-1)^n}{3^n}$.

Answer:

(b) (3 points) $\sum_{n=1}^{\infty} \left(\frac{2n^2+n}{4n^2-3}\right)^n$.

Answer:

(c) (3 points) $\sum_{n=1}^{\infty} \frac{2n^2+n}{4n^2-3}$.

Answer:

(d) (3 points) $\sum_{n=3}^{\infty} \frac{\ln(n)}{n}$.

Answer:

(e) (3 points) $\sum_{n=1}^{\infty} \ln(n) - \ln(n+1)$.

Answer:

Problem 4 (8 points) Consider the power series

$$\sum_{n=1}^{\infty} \frac{1}{5^n \cdot n^4} x^n.$$

(a) (4 points) Determine the radius of convergence of this power series.

Answer:

(b) (4 points) Determine the interval of convergence of this power series.

Answer:

Problem 5 (10 points)

(a) (4 points) Using either the Maclaurin series for $\sin(x)$ on the cover page or the definition of Maclaurin series, determine the Maclaurin series for $f(x) = \cos(x)$.

Answer:

(b) (3 points) Using your answer to (a), find the Maclaurin series for $g(x) = \cos(2x^2)$.

Answer:

(c) (3 points) Using your answer to (b), find the value of $g^{(12)}(0)$ (the twelfth derivative of g at 0).

Answer:

Problem 6 (10 points)

(a) (3 points) Sketch the graph of the polar curve $r = 2\cos(5\theta)$.

Answer:

(b) (7 points) Set up and evaluate an integral to find the area enclosed by this curve.

Answer:

Problem 7 (10 points)

(a) (3 points) Use the Maclaurin series of $\sin(x)$ to find the Maclaurin series of $f(x) = x\sin(3x)$.

Answer:

(b) (3 points) Use the binomial theorem to find the Maclaurin series of $g(x) = 1/\sqrt{1+x^2}$.

Answer:

(c) (4 points) Use your answers from (a) and (b) to evaluate

$$\lim_{x\to 0} \frac{3x^2 - x\sin(3x)}{-1 + \frac{x^2}{2} + \frac{1}{\sqrt{1+x^2}}}.$$

Answer:

Problem 8 (8 points) Use Euler's method with stepsize 0.1 to approximate $y(0.2)$, where y is the solution to

$$y' = x + y$$

with initial condition $y(0) = -1$.

Answer:

Problem 9 (8 points) Find the solution to the differential equation

$$y' = e^y + x^2 e^y$$

satisfying the initial condition $y(0) = 0$.

Answer:

Problem 10 (8 points) Find the general solution to the following differential equation

$$xy' + 3y = \frac{\sin(x)}{x}.$$

Answer: