Mathematics UN1102
Section 3, Fall 2017
Practice Midterm 2
November 15, 2017
Time Limit: 75 Minutes

Name: ______________________________

UNI: ______________________________

Instructions: This exam contains 6 problems. Please make sure you attempt all problems.

Present your solutions in a **legible, coherent** manner. Unless otherwise specified, you should show your work; you will be evaluated on both your reasoning and your answer. Unsupported or illegible solutions may not receive full credit.

Please write your **final answer** for each problem in the provided box. Please show your work in the space below the box. If you need additional space for scratchwork, you may use the blank pages stapled to the end of the exam. Please **do not write on the back side of pages**.

The use of outside material including books, notes, calculators, and electronic devices is not allowed.

Question	1	2	3	4	5	6	Total
Points	15	10	20	15	20	20	100
Score							

Formulas

Maclaurin series:

- $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots$ $\quad R = 1$
- $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ $\quad R = \infty$
- $\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ $\quad R = \infty$
- $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ $\quad R = 1$
- $(1+x)^k = \sum_{n=0}^{\infty} \frac{k(k-1)\cdots(k-n+1)}{n!} x^n = 1 + kx + \frac{k(k-1)}{2!} x^2 + \frac{k(k-1)(k-2)}{3!} x^3 + \cdots$ $\quad R = 1$

Problem 1 (15 points) In each of the following questions you will be asked to give examples of sequences or series that satisfy certain properties, or explain why no such examples exist.

(a) (5 points) Write down an example of a sequence $\{a_n\}$ such that

- $\{a_n\}$ is divergent;
- $a_n < 5$ for all n, and;
- $a_n < a_{n+1}$ for all n;

or explain why no such divergent sequence exists.

Answer:

(b) (5 points) Write down two divergent series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ such that the series $\sum_{n=1}^{\infty} a_n b_n$ is convergent, or explain why no such pair of series exists.

Answer:

(c) (5 points) Write down an example of a convergent series $\sum_{n=1}^{\infty} a_n$ such that

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 1,$$

or explain why no such convergent series exists.

Answer:

Problem 2 (10 points) Determine whether the following sequences are convergent or divergent. If the sequence is convergent, determine its limit as $n \to \infty$. Justify your answer.

(a) (5 points) The sequence $\{a_n\}$, where $a_n = \ln\left(\frac{n^2+3n}{7n^2+4}\right)$.

Answer:

(b) (5 points) The sequence $\{b_n\}$, where $b_n = \frac{4^n}{n!}$.

Answer:

Problem 3 (20 points) Determine whether the following series are convergent or divergent. If the series is convergent, determine its sum.

(a) (10 points) The series $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n+1}} - \frac{1}{\sqrt{n}} \right)$.

Answer:

(b) (10 points) The series $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3+10}}$.

Answer:

Problem 4 (15 points) Consider the series $\sum_{n=1}^{\infty} ne^{-n^2}$.

(a) (5 points) Check that this series satisfies the hypotheses of the integral test.

Answer:

(b) (10 points) Use the integral test to determine whether this series is convergent or divergent. You **do not** need to determine the sum if convergent.

Answer:

Problem 5 (20 points) Write down a power series $\sum_{n=0}^{\infty} c_n x^n$ whose interval of convergence is $(-1, 1]$, and show why the power series you wrote down has interval of convergence $(-1, 1]$.

Answer:

Problem 6 (20 points)

(a) (10 points) Find the Maclaurin series for the function

$$f(x) = \frac{1}{\sqrt{1-x^2}}.$$

Answer:

(b) (10 points) Using your answer above, find the Maclaurin series for the function

$$g(x) = \sin^{-1}(x).$$

Hint: Recall that $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}(x) + C$.

Answer: