Mathematics UN1102   Name: _____
Section 1, Spring 2020
Midterm 1 Practice Exam A   UNI: _____
Time Limit: 75 Minutes

Instructions: This exam contains 7 problems. Please make sure you attempt all problems.

Present your solutions in a **legible, coherent** manner. Unless otherwise specified, you should show your work; you will be evaluated on both your reasoning and your answer. Unsupported or illegible solutions may not receive full credit.

Please write your **final answer** for each problem in the provided box. Please show your work in the space below the box. If you need additional space for scratchwork, you may use the back side of the problem sheets or the blank pages stapled to the end of the exam.

The use of outside material including books, notes, calculators, and electronic devices is not allowed.

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Formulas

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$$\sin^2\theta + \cos^2\theta = 1 \qquad\qquad\qquad 1 + \tan^2\theta = \sec^2\theta$$

$$\sin(2\theta) = 2\sin\theta\cos\theta \qquad\qquad\qquad \cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\sin^2\theta = \frac{1}{2}(1 - \cos(2\theta)) \qquad\qquad\qquad \cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C \qquad\qquad \int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \arcsin\left(\frac{x}{a}\right) + C$$

$$\int \tan x\,dx = \ln|\sec x| + C \qquad\qquad\qquad \int \sec x\,dx = \ln|\sec x + \tan x| + C$$

Problem 1 (10 points) Evaluate the definite integral

$$\int_0^1 xe^x dx.$$

Answer:

Problem 2 (15 points) Using an appropriate trigonometric substitution, evaluate the indefinite integral

$$\int \frac{\sqrt{x^2 - 9}}{x^4} dx.$$

Answer:

Problem 3 (15 points) Evaluate the indefinite integral

$$\int \theta \cos^2 \theta \, d\theta.$$

Answer:

Problem 4 (20 points) Evaluate the indefinite integral

$$\int \frac{x^2 + 4x - 3}{x^3 + x^2 + x + 1} dx$$

in the following two steps.

(a) (10 points) Write a partial fraction decomposition for

$$\frac{x^2 + 4x - 3}{x^3 + x^2 + x + 1}.$$

Answer:

(b) (10 points) Evaluate the resulting integral.

Answer:

Problem 5 (15 points) Does the improper integral

$$\int_1^\infty \frac{1}{x^2 + 1}\,dx$$

converge or diverge? If it converges, compute its value.

Answer:

Problem 6 (20 points) Consider the region A between the line $y = x$ and the curve $y = x^4$.

(a) (5 points) Find the two points where the two graphs intersect, and sketch the region between the two graphs.

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Answer:

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(b) (15 points) Compute the volume of the solid of revolution obtained by rotating A about the y-axis. State whether you are using the method of disks/washers or the method of cylindrical shells.

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Answer:

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Problem 7 (5 points) Set up a definite integral to compute the arc length of the curve

$$y = \sin(x), \qquad \frac{\pi}{2} \le x \le \pi.$$

You **do not** need to evaluate the integral.

Answer: